de Sitter Radiation and Backreaction in Quantum Cosmology

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Abstract

We explore the quantum cosmology description of the de Sitter (dS) radiation and its backreaction to dS space, inherent in the wave function of the Wheeler-DeWitt equation for pure gravity with a cosmological constant. We first investigate the quantum Friedmann-Lemaitre-Robertson-Walker cosmological model and then consider possible effects of inhomogeneities of the universe on the dS radiation. In both the cases we obtain the modified Friedmann equation, including the backreaction from spacetime fluctuations, and the quantum-corrected dS temperature. It is shown that the quantum correction increases the dS temperature with the increment characterized by the ratio of the dS scale to the Planck scale.

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Introduction.— The de Sitter (dS) space is essential in cosmology and in the study of the quantum aspects of gravity. In cosmology the inflation, the early-time accelerating expansion that is nearly dS, plays an important role in giving the initial big bang and the favorable initial condition of our universe. In particular it generates the seeds of the cosmic structures, meanwhile giving a rather flat, homogeneous and isotropic universe that is simple to study. In addition, the late-time accelerating expansion may eventually approach dS, since the ΛCDM model with a positive cosmological constant for driving the cosmic acceleration fits the current observational results well.

The dS radiation, as the Hawking radiation for black holes, entails quantum theory since it is a consequence of the unitary inequivalence of the in-vacuum and the out-vacuum for quantum field excitations. Though the dS radiation can be understood within the quantum field theory on a classical spacetime, it may also be studied in a more fundamental framework, quantum cosmology or quantum gravity. One may thus raise a question about the physics involved in the quantum cosmological model for the dS radiation. In quantum cosmology prescribed by the Wheeler-DeWitt (WDW) equation from the Hamiltonian formulation of general relativity [1], the universe is described by a wave function of the spacetime geometry and the matter fields, and so is the dS radiation. Accordingly, the dS radiation, as well as its backreaction to dS space, should be inherent in the quantum cosmology description of the dS universe.

The quantum correction to the gravitational field equations in quantum cosmology has been investigated by Banks [2] and further elaborated by Brout et al [3, 4]. The WDW equation for gravity coupled to matter fields has two mass scales: the Planckian mass scale for the gravity part and the mass or energy scale for the matter fields. Therefore, the WDW equation may have the Born-Oppenheimer approximation, used for an atom of heavy nucleus and electrons, such that the heavy gravitational part separates from the light matter part in the approximate solution for the WDW equation and the gravitational part leads to the semiclassical Einstein equation as the Hamilton-Jacobi equation [2–4]. However, the Banks-Born-Oppenheimer (BBO) approximation cannot be employed for pure gravity with a cosmological constant because the gravity is the only scale in the WDW equation. In this case, one may use the de Broglie-Bohm interpretation of the wave function, in which the oscillatory wave packet is peaked along a new trajectory governed both by the classical potential and the quantum potential [5, 6]. The dS spacetime emerges from the

oscillatory wave packet. Further the motion of matter in imaginary time in classically forbidden configurations of gravity can be related to an inverse temperature [3, 4].

In this paper we investigate the dS radiation and its backreaction in quantum cosmology with pure gravity, i.e., where we consider solely the spacetime metric in the field content and explore the intrinsic dS radiation as the gravitational fluctuations. We will firstly investigate the case with a homogeneous and isotropic, i.e. simply time-dependent, metric. We will then consider the possible effects from the spatial inhomogeneities of the metric. In both the cases we obtain the modified Friedmann equation and thereby the modified dS temperature with the quantum correction from the wave function that encloses the backreaction of the dS radiation. For comparison, in the following we start with the framework of the quantum field theory on classical spacetime before presenting our quantum cosmology analysis.

de Sitter radiation in classical spacetime.— Here we treat dS space as a classical background and disregard the backreaction of the dS radiation. Since Gibbons and Hawking proposed the dS radiation [7] soon after discovering the black hole radiation [8], there have been several derivations of the dS temperature presented [9–11]. In the following we invoke the correspondence between the inverse temperature and the Euclidean time period for the derivation. We consider the classical Friedmann equation in the Euclidean time for a closed universe ($k = 1$) in unit of $c = \hbar = 1$,

$$-\left(\frac{a'}{a}\right)^2 + \frac{k}{a^2} = \frac{\Lambda}{3} \equiv H_\Lambda^2 \,, \tag{1}$$

where the prime is the derivative with respect to the Euclidean time and Λ is the cosmological constant. This equation gives the inverse temperature as a period [3, 4]

$$\beta_0 \equiv \frac{1}{k_B T_0} = 2\int_{-1/H_\Lambda}^{1/H_\Lambda} \frac{da}{\sqrt{1 - H_\Lambda^2 a^2}} = \frac{2\pi}{H_\Lambda} \,. \tag{2}$$

As a result, the dS temperature is given by

$$T_0 = \frac{H_\Lambda}{2\pi k_B} \,. \tag{3}$$

de Sitter radiation in quantum cosmology with homogeneous metric perturbations.— Now we consider a system of pure gravity and a cosmological constant with the action

$$S = -\frac{1}{16\pi G}\int d^4x \sqrt{-g}\,(R - 2\Lambda) \,. \tag{4}$$

In the following we derive the modified Friedmann equation [12, 13] and thereby the modified dS temperature from the WDW equation for a homogeneous and isotropic universe described by the closed Robertson-Walker metric,

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega_3^2 \,. \tag{5}$$

Substituting this metric into the action gives

$$S = -\frac{3}{8\pi G}\int dt\, a^3\left[\frac{\dot{a}^2}{Na^2} + N\left(-\frac{k}{a^2} + \frac{\Lambda}{3}\right)\right], \tag{6}$$

where the overdot denotes the time derivative. The WDW equation from the super-Hamiltonian constraint for the Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological model reads

$$\left[-\frac{1}{2}G\frac{\partial^2}{\partial a^2} + \frac{9}{32\pi^2 G}a^4\left(\frac{k}{a^2} - \frac{\Lambda}{3}\right)\right]\Psi(a) = 0\,, \tag{7}$$

where $\Psi(a)$ is the wave function of the universe.

We follow the de Broglie-Bohm interpretation for an oscillatory wave packet [5, 6], in which the wave packet is peaked along a trajectory guided not only by the classical potential but also by the quantum potential. For that purpose, one writes

$$\Psi(a) = F(a)\exp[iS(a)], \tag{8}$$

and obtains the semiclassical Einstein equation from the real part of the WDW equation, i.e. the Hamilton-Jacobi equation:

$$\frac{1}{2}G\left(\frac{\partial S}{\partial a}\right)^2 + \frac{9}{32\pi^2 G}a^4\left(\frac{k}{a^2} - \frac{\Lambda}{3}\right) + V_q(a) = 0\,, \tag{9}$$

where the quantum potential

$$V_q(a) = -\frac{G}{2F}\frac{\partial^2 F}{\partial a^2}\,. \tag{10}$$

On the other hand, the imaginary part of the WDW equation gives the continuity equation of the probability,

$$F\frac{\partial^2 S}{\partial a^2} + 2\frac{\partial F}{\partial a}\frac{\partial S}{\partial a} = 0\,. \tag{11}$$

One may define the cosmological time via parameterizing the de Broglie-Bohm trajectory along the tangential direction of the gravitational action [2–4]:

$$\frac{\partial}{\partial t} = -\frac{4\pi G}{3a}\frac{\partial S}{\partial a}\frac{\partial}{\partial a}\,. \tag{12}$$

With this definition of time, the semiclassical Einstein equation gives the modified Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\Lambda}{3} - \frac{32\pi^2 G}{9a^4} V_q \,, \tag{13}$$

where the quantum potential for quantum corrections can be given by inverting Eq. (11) [12, 13]

$$V_q = -\frac{G}{2}\left[\left(\frac{(a\dot{a})^{\cdot}}{2a\dot{a}^2}\right)^2 - \frac{1}{2\dot{a}}\left(\frac{(a\dot{a})^{\cdot}}{a\dot{a}^2}\right)^{\cdot}\right]. \tag{14}$$

A few remarks are in order. Firstly, the origin of quantum potential and thereby quantum corrections is quantum fluctuations of the spacetime geometry, being the scale factor $a(t)$ here. Thus the WDW equation in this case describes only the homogeneously and isotropically fluctuating geometry. Note that massless scalar fields under the symmetry of homogeneity and isotropy are stiff matter with the equation of state $w = 1$, and contribute to the modified equation (13) as

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\Lambda}{3} - \frac{32\pi^2 G}{9a^4} V_q + \frac{4\pi G}{3a^6}\sum_{i=0}^{n} p_i^2 \,, \tag{15}$$

where p_i is the momentum of the i-th scalar field. Secondly, due to the quantum potential in the modified Friedmann equation, the classical dS metric is no longer a solution. It is extremely difficult to exactly solve the nonlinear differential equation (13). Nevertheless, when the quantum corrections are much smaller than the cosmological constant, the scale factor $a(t)$ may still approximately follow the dS behavior. Under this approximation we have

$$V_q \simeq -\frac{G}{a^2} \,, \tag{16}$$

and the modified Friedmann equation reduces to

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} \simeq H_\Lambda^2 + \frac{c_6}{a^6} \,, \quad c_6 \equiv \frac{32\pi^2}{9} G^2 \,. \tag{17}$$

It is interesting to note that the quantum correction under the dS approximation is proportional to a^{-6} and therefore behaves like a stiff matter as the homogeneous and isotropic massless scalar fields in Eq. (15).

With the Friedmann equation (17) we obtain the dS temperature via the previously demonstrated method invoking the Euclidean time period. Thus the inverse temperature is given by

$$\beta_1 \equiv \frac{1}{k_B T_1} = \frac{2}{H_\Lambda}\left(\int_{-\tilde{a}_1}^{-\tilde{a}_0} + \int_{\tilde{a}_0}^{\tilde{a}_1}\right)\frac{d\tilde{a}}{\sqrt{1 - \tilde{a}^2 - (\tilde{c}_6/\tilde{a})^4}} \,, \tag{18}$$

where

$$\tilde{a} \equiv H_\Lambda a \,, \quad \tilde{c}_6 \equiv H_\Lambda c_6^{1/4} \sim \frac{H_\Lambda}{M_{\rm pl}} \quad (M_{\rm pl}\text{: Planck scale}), \tag{19}$$

are the dimensionless quantities and $\tilde{a}_0$ and $\tilde{a}_1$ are the only two positive roots of the denominator of the integrand. When $\tilde{c}_6 \ll 1$,

$$T_1 \simeq \frac{H_\Lambda}{2\pi k_B} \left\{ 1 + \frac{\tilde{c}_6}{4\sqrt{2}\, 3^{3/4}\, \Gamma\left(5/4\right)^2} + \mathcal{O}\left[\tilde{c}_6^2\right] \right\}. \tag{20}$$

We note that the leading correction proportional to $\tilde{c}_6$ is positive and therefore $T_1 > T_0$. Consequently the quantum correction makes the dS temperature higher than that of classical spacetime in Eq. (3).

de Sitter radiation in quantum cosmology with inhomogeneous metric perturbations.— The dS radiation from the inhomogeneous perturbations contribute to the Friedmann equation a relativistic energy density ($\propto a^{-4}$) as follows,

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} \simeq H_\Lambda^2 + \frac{c_4}{a^4} + \frac{c_6}{a^6}\,, \tag{21}$$

where c_4 is an undetermined constant related to the abundance of the dS radiation from inhomogeneous perturbations. This can be read from Ref. [14] where Halliwell and Hawking investigated the inhomogeneous or isotropic modes of a closed FLRW universe in the framework of quantum cosmology. In the WDW equation, in addition to the second-derivative terms deduced from the conjugate momentum-square of these modes, the spatial variation of these modes contributes the "mass" terms that are proportional to a^4 and to the field-strength-square (see Eqs. (5.30)–(5.32) in Ref. [14]). For example, regarding the mode of the standard scalar harmonics on a three-sphere with a_{nlm} as the amplitude or the field strength of the mode, in the WDW equation this mode contributes a second-derivative term $\partial^2/\partial {a_{nlm}}^2$ from its conjugate momentum and a "mass" term $-a^4(n^2-5/2)a_{nlm}^2/3$ from its spatial variation. Such mass terms will eventually contribute to the Friedmann equation an energy term proportional to a^{-4} like the c_4 term in Eq. (21).

Via the Euclidean time period method, the Friedmann equation (21) gives the inverse dS temperature:

$$\beta_2 \equiv \frac{1}{k_B T_2} = \frac{2}{H_\Lambda}\left(\int_{-\tilde{a}_1^*}^{-\tilde{a}_0^*} + \int_{\tilde{a}_0^*}^{\tilde{a}_1^*}\right) \frac{d\tilde{a}}{\sqrt{1-\tilde{a}^2-(\tilde{c}_4/\tilde{a})^2-(\tilde{c}_6/\tilde{a})^4}}\,, \tag{22}$$

where $\tilde{a}_{0,1}^*$ are the only two positive roots of the denominator of the integrand, and the dimensionless constant $\tilde{c}_4 \equiv H_\Lambda c_4^{1/2}$. Consider two limiting cases: c_6 domination and c_4

domination, regarding the quantum correction to the Friedmann equation. Since the c_6 correction decreases faster than the c_4 correction along with the cosmic expansion, at earlier times the c_6 correction may dominate over c_4 and later c_4 dominates. When c_6 dominates, i.e., when the contribution from the homogeneous perturbation to the dS radiation dominates, we find $T_2 \simeq T_1$. In contrast, later, when the contribution from inhomogeneous perturbations dominates (c_4 domination), the dS temperature approaches that of a classical spacetime: $T_2 \simeq T_0$. We particularly note that, when $c_6 = 0$, we have $T_2 = T_0$, regardless of the value of c_4, i.e., regardless the abundance of the radiation. Thus, regarding the backreaction of the dS radiation and the resultant quantum correction to the dS temperature, the c_4-type radiation alone does not change the dS temperature. Between these two limiting cases, $T_0 < T_2 < T_1$. As a result, along with the cosmic expansion, the dS temperature decreases from T_2 to T_0.

Here we compare our work with the studies of the backreaction of the dS radiation in the framework of a classical spacetime. For example, Ref. [15] investigates the backreaction of the spherically symmetric dS radiation to dS space in the Painlevé coordinates and finds that the dS temperature decreases after the emission of the dS radiation, with the decrement proportional to the energy of the radiation and to $(H_\Lambda/M_{\rm pl})^2$. For comparison, we note that in our work the dS radiation is intrinsic quantum fluctuations of gravity that exist at all times but no process of dS radiation emission is involved. Along with the cosmic expansion, the temperature of such purely gravitational dS radiation decreases to $H_\Lambda/(2\pi)$ from a higher value with the temperature difference that is caused by quantum corrections and characterized by $H_\Lambda/M_{\rm pl}$.

Summary.— We investigate the dS radiation and its backreaction to dS space in quantum cosmology where the dS radiation is expected to be inherent in the wave function of the universe. We consider the simple system with pure gravity and a positive cosmological constant. We have shown that the dS radiation from the homogeneous perturbation contributes to the modified Friedmann equation an energy density behaving like stiff matter, while the dS radiation from inhomogeneous perturbations may contribute a relativistic energy density. We have shown that the backreaction of the dS radiation increases the dS temperature in the case with a significant contribution from the homogeneous perturbations. In contrast, the dS radiation from inhomogeneous perturbations barely changes the dS temperature, an interesting feature we particularly note. The increment of the dS temperature is charac-

terized by $H_\Lambda/M_{\rm pl}$, the ratio of the dS scale to the Planck scale. Thus, when the dS scale approaches the Planck scale, the temperature increment induced by the quantum correction is significant. On the other hand, along with the cosmic expansion the quantum correction is more and more insignificant and the dS temperature decreases and approaches to $H_\Lambda/(2\pi)$, the temperature of a classical dS space.

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